

A model which displays second law behavior can be constructed using an approach similar to that used by Brian Zhang to derive the energy distribution. The observation that the coconuts (energy quanta) can be described as bosons suggests that the islanders (molecules) could be modelled as fermions, since no two molecules can occupy the same space. If we divide the space in the box into cubes the dimension of the diameter of the molecules, the number of these cubes will be about 1000 per molecule, since the average distance between molecules at room temperature averages about 10 times their diameter. The molecules are assumed to interact according to a Markov process where they can move one cube in the same direction according to the law of inertia, if not obstructed by another molecule. Otherwise they will be randomly assigned to one of the six directions, simulating a collision. If there is another molecule in the adjacent cube toward which the molecule is directed, the molecule does not move and the cycle is repeated. Since this model incorporates a crude version of Newton's laws, it demonstrates that the laws of motion are not time symmetric, resolving Loschmidt's paradox.

If the gas is initially confined to, say, the left half of the box and then released, the early motion will be biased toward the right, since there are no molecules to constrain movement in that direction. As more and more molecules fill the rest of the box, this bias will continue due to the lower density of molecules on the right. At some point the density will become roughly uniform and an equilibrium state will be reached where the average direction of motion of the molecules will be unbiased. Like Zhang's energy model, we have a physical model that does not depend on macrostate probabilities. Following Zhang, we consider the

molecules indistinguishable. In this case the number of microstates is counted by

$$\Omega_{Tot} = \frac{M!}{N! (M - N)!}$$

For the code instantiation of this logic for two-dimensional space and 8 directions, see `SpatialLeftOct.py`. This generates a time series of the number of molecules in the right half of the box, as exemplified by `RunLeftOct100.xlsx`, which plots the results of a system with 100 molecules. Note that finding exactly 50 molecules in the right half of the box is a rarity.

The time series also exhibits periodicity. In the case of a square box with the length of a side set to 100, the wavelength increases linearly with the number of molecules, starting from a minimum value of 200. This is illustrated by `Wavelength.xlsx`.

This algorithm can be modified to run Zang's coconut exchange code whenever a collision occurs. The resulting algorithm provides a complete model of the evolution of an ideal gas toward equilibrium.

The fermion model was known to Boltzmann, since Rudolf Clausius in 1857 had estimated the ratio of the volume of space occupied by a molecule to the volume of the molecule's "sphere of action". He came up with an estimate of 1000, which is the same as mine. Boltzmann gives a similar example on page 95 of his *Lectures on Gas Theory*, where he calculates the ratio of the volume containing a certain number of molecules in a gas to the volume containing the same number of molecules in a liquid as 813. Since the actual volume occupied by these molecules is less

than the volume of the liquid containing the same number of molecules, the ratio of the volume occupied by a gas to the volume occupied by the molecules is comparable to Clausius' estimate.

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